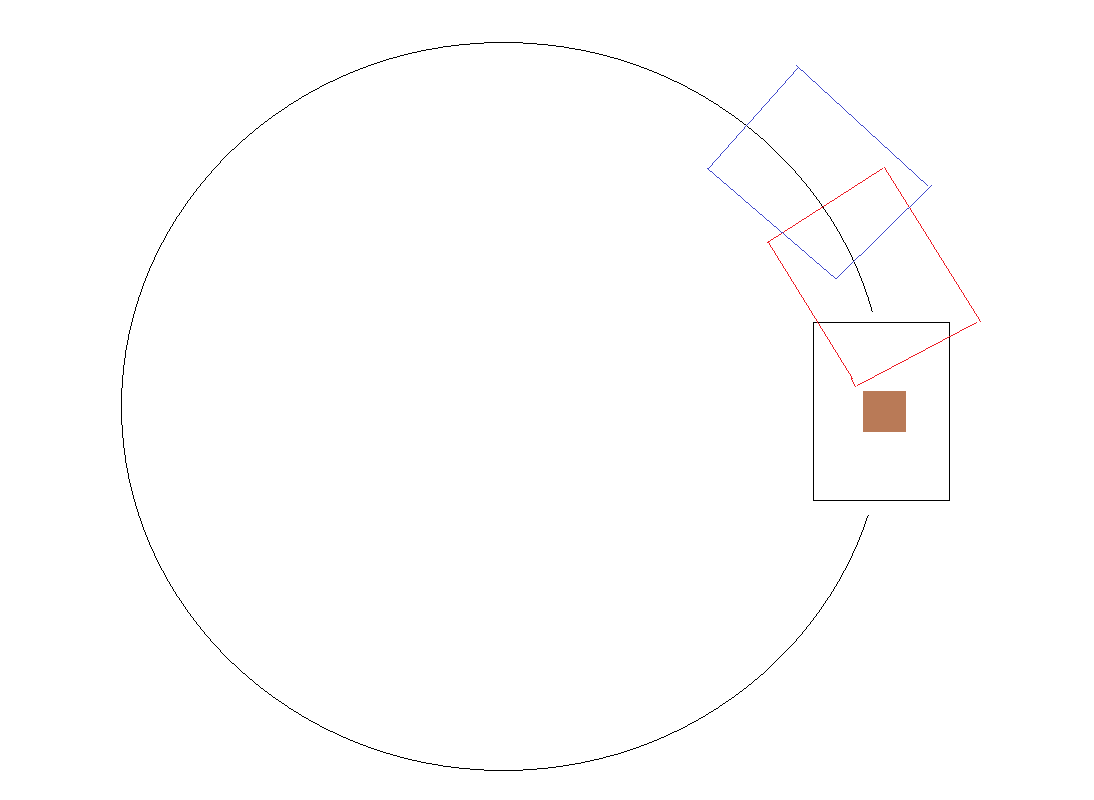
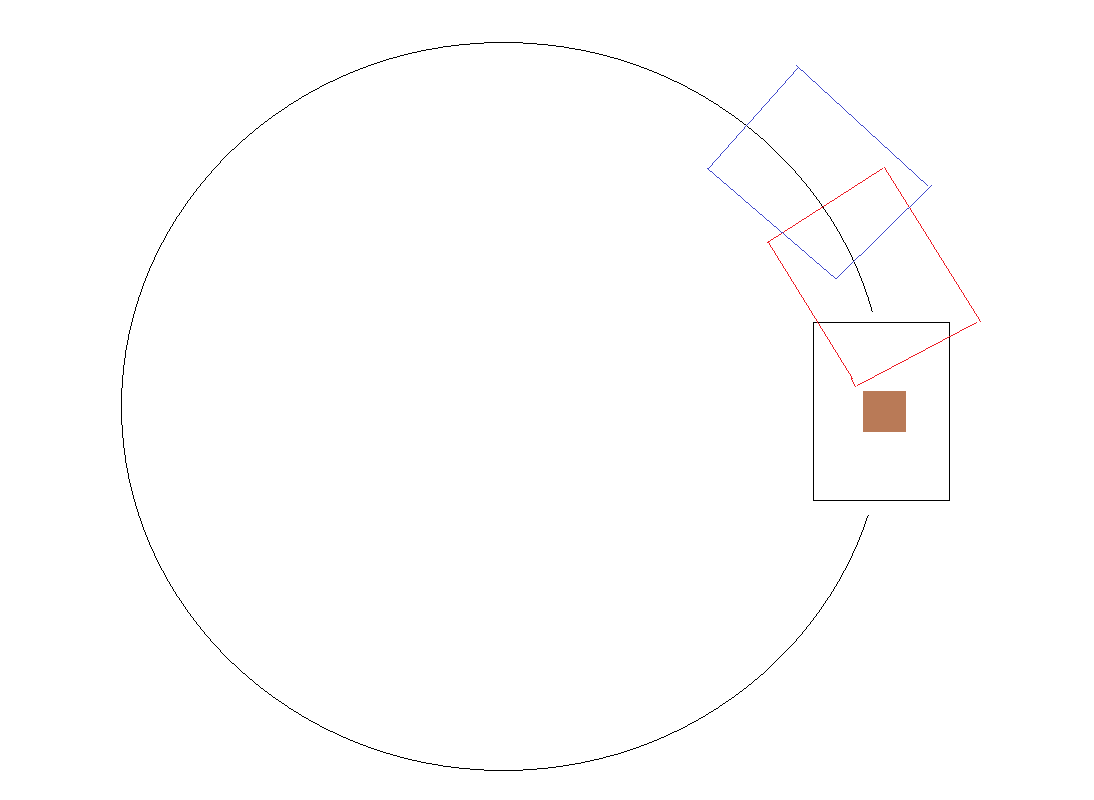
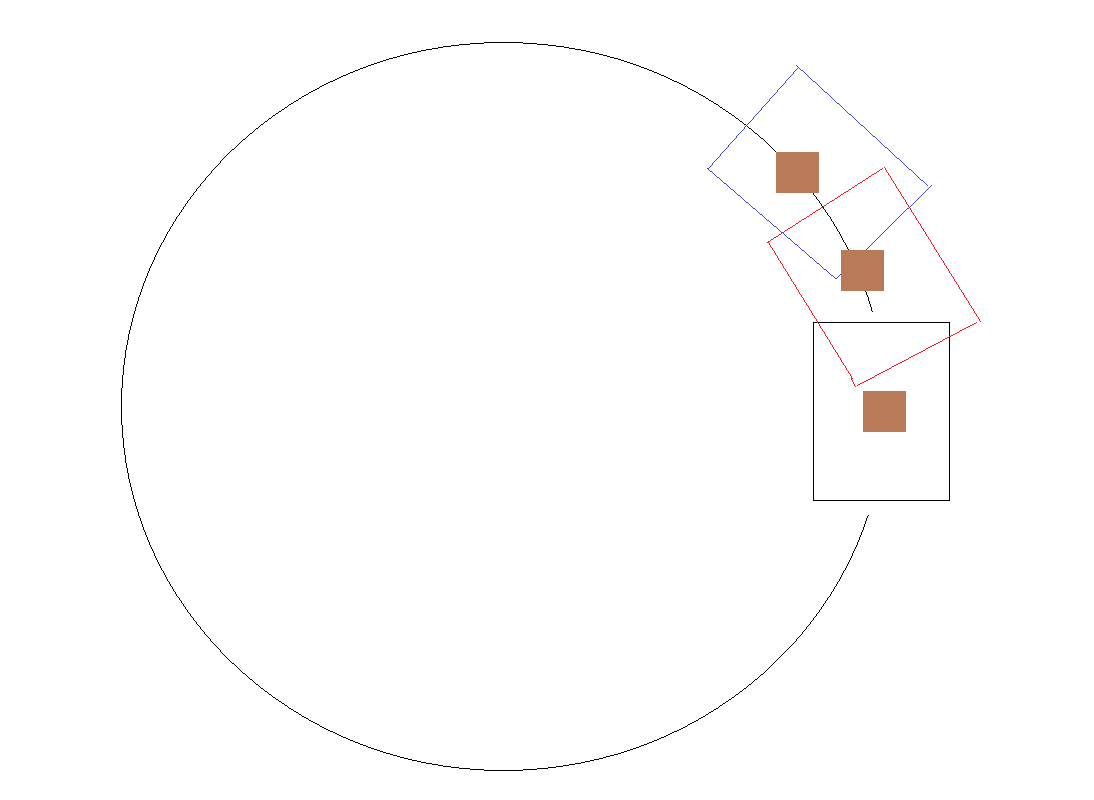
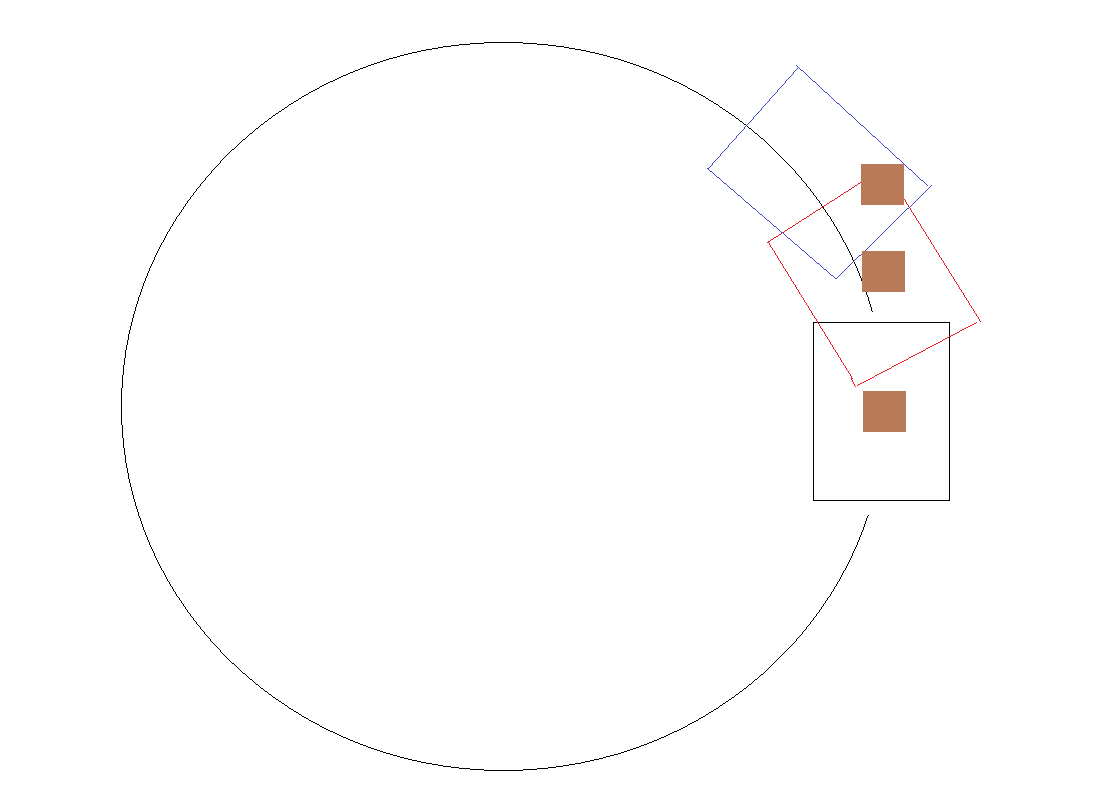
**Homework 6 Solutions due 3/1**

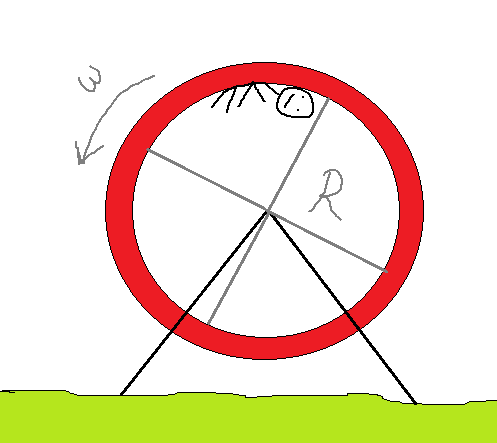
\* In all problems below, you may neglect air resistance and the rolling friction force.

**Problem 1.** Say you’re in a car rounding a curve, with a bag of groceries in the front seat. (a) Suppose the friction force between the bag and the seat is high enough to keep it from slipping. On the left, draw (please) the position of the bag at each of the two later times. (b) Suppose friction is too small, allowing the bag to slip. Then on the right, draw the bag’s position at the two later times.

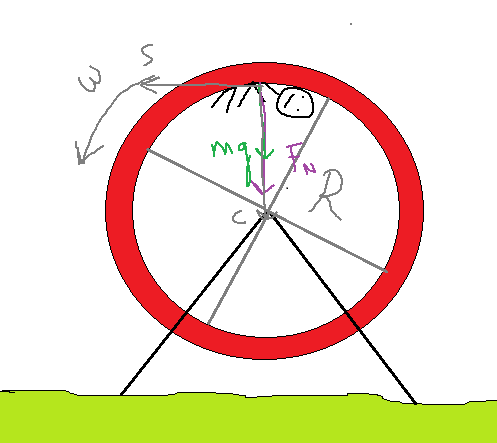
 

**Problem 2.** Suppose you were riding a vertical gravitron, instead of a horizontal one. Let the radius be R = 5m.



(a) The gravitron is presently rotating at ω = 1 rad/s. Will this suffice to keep the person in contact with the wall, at the top? Hint: calculate the normal force. And even though you won’t know what it is exactly, because you don’t know the mass, you’ll know the sign of the force, which is enough.





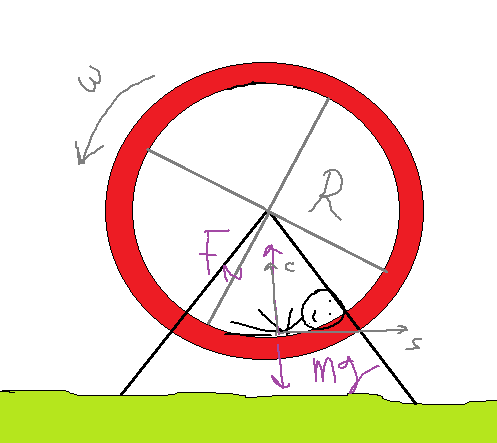
A negative normal force would mean the force is pointing up, but this cannot happen, unless the surface were glue or something, so this rate is not sufficient.

(b) What is the minimum rate of rotation necessary to keep the person at the top?

Repeating the analysis, and setting FN = 0, we’d have:



(c) If the rate of rotation is ω = 2 rad/s, by what factor is their apparent weight greater than their actual weight, at the bottom of their rotation?



Again, this is the normal force,



The ratio of this to their actual weight is:



**Problem 3.** Say you’re driving along in your car with a bag of groceries sitting on the passenger side seat. The coefficient of static friction between the seat and groceries is μs = 0.75.

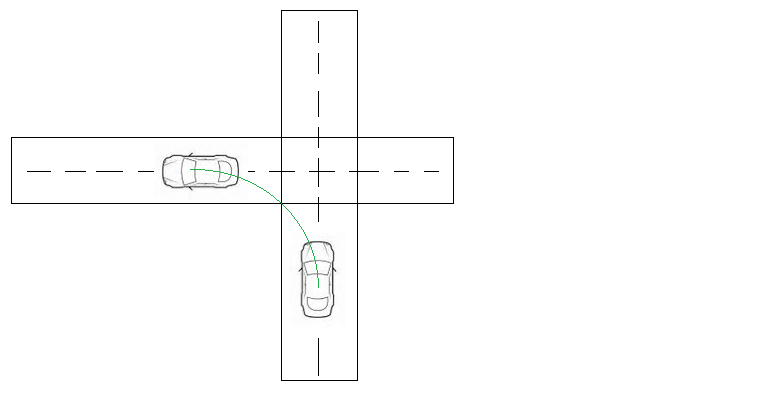
(a) You see a cute turtle trying to cross the road in front of you; so you slam on the breaks, decellerating from 20m/s to 0m/s in 3s. Do the groceries stay on your seat, or slide off? Please justify your answer ☺. And this doesn’t pertain to circular motion.

We can compare the acceleration the groceries would have, to the maximum acceleration that friction could impart.



So the magnitude of a is less than the magnitude of amax. So the groceries will not slip.

(b) The groceries are on your seat again. This time, you’re rounding the turn into your subdivision. The distance you travel along the green path is 30m, at a speed v = 10m/s. Do the groceries slide on your seat? Please justify.



The acceleration the groceries experience in this case is:

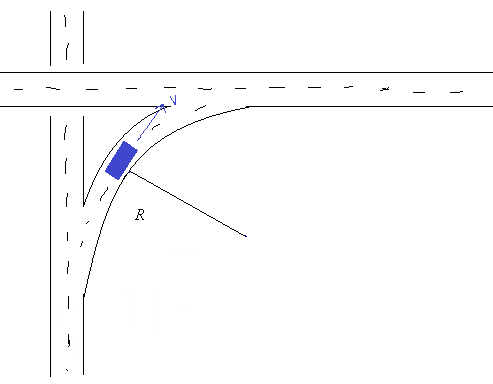
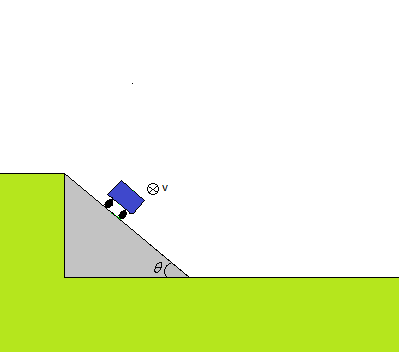


The radius of the circular path can be calculated by recognizing the path as a quarter circle, which implies a quarter circumference = 30m → (1/4)(2πR) = 30 → R = (2/π)∙30 = 19.1 m. So filling this in,

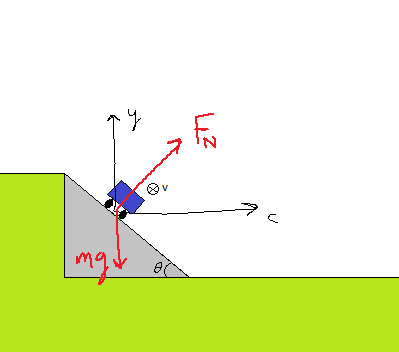


This also is less than amax, so the groceries stay on once more. You’re a safe driver ☺. I don’t think my groceries ever *fail* to slip.

**Problem 4.** On-ramps are often banked so that you can turn at highway speeds, even if oil + rain should significantly reduce the friction force, which is what would normally be responsible for keeping you on the circular path. A top-down, and side view are depicted below. Let the radius of curvature of the on-ramp be 600m. Then assuming friction is absent, what minimum banking angle would allow you to maintain a velocity of v = 35m/s? Angle is exaggerated a bit/lot in the diagram.

Forces and axes are shown below. Note the ‘s’ axis points into the page, but isn’t shown.



Applying N2L in the y direction, we have:



And in the c direction:



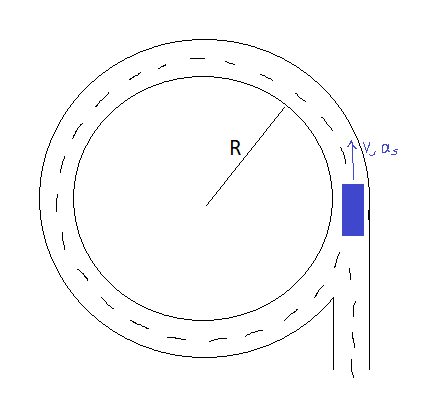
Then plugging the normal force into this expression,



And so,

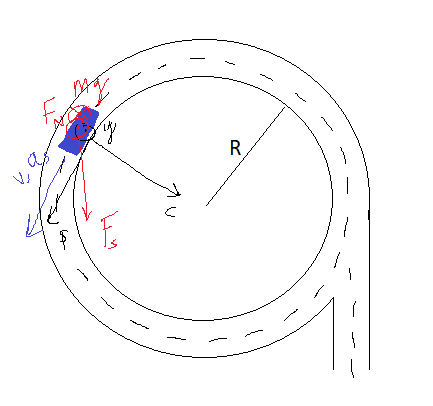


**Problem 5.**  Like that problem in class, suppose a car enters a horizontal circular track with speed v = 0m/s, and acceleration as = 3m/s2. Let the radius of the track be R = 100m, and the coefficients of friction between the track and tires are μs = 0.50, and μk = 0.30. You may ignore rolling friction, and air resistance, as usual.



(a) At what speed will the car start to slip? Note you don’t need to know the mass of the car. Just leave it as a symbol, and it will cancel out eventually. And also, since the car is accelerating, friction will have both a tangential, and centripetal component (like in class).

It will slip when the friction force equals it’s maximum value, μ­sFN. So drawing forces, and filling stuff into N2L, we have:





Now we fill in what we know of the accelerations:



And then we construct the static friction force,



and set it equal to the maximum value,



And then we solve for the velocity,



(b) Starting time from when the car enters the track, *when* does it start to slip?

This is when



(c) What is its arc length at this time? What angle does this correspond to?

Arc length would be:

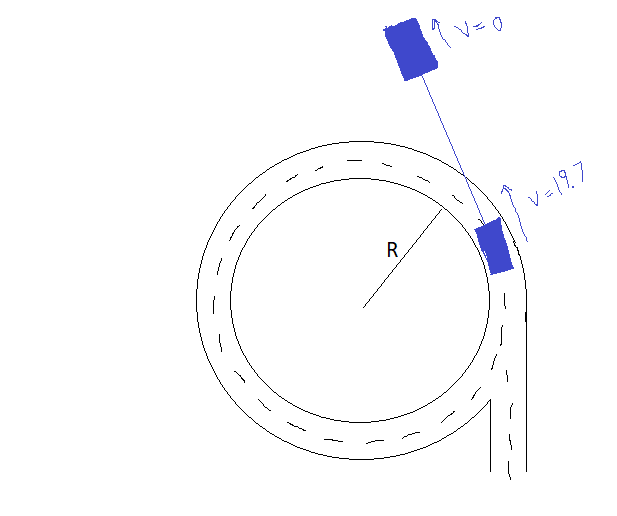


Corresponding angle is:



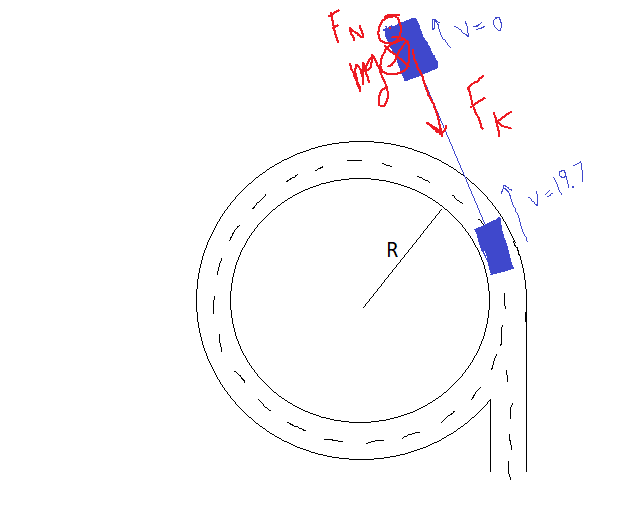
(d) Draw where the car is when it starts to slip, and what its path will be thereafter.

Will look like this:



(e) How long will it slide before coming to rest?

Must find the acceleration. Now its,

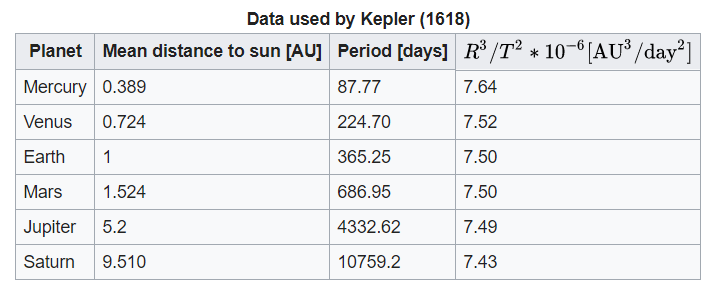




This is against the direction of motion. So the time will be given by:



**Problem 6.** In the early 17th century, Johannes Keppler, discovered a pattern between a planet’s oribtal period (time it takes for it to orbit the Sun), T, and the planet’s mean distance from the Sun, R. This was his approximate data. Note 1 AU (astronomical unit) = 1.5×1011m.



As you can see, the ratio R3/T2 appears to be constant (it’s not exactly, but that’s mainly because Kepler’s data wasn’t as accurate as the data we have now). About 70 years later, Isaac Newton was able to predict that this ratio *should* be constant, and moreover, to provide a theoretical estimate for what the ratio should be, which very closely matched Kepler’s data. This provided great support for his gravitational field formula g = GM/r2.

(a) Starting from ac = ΣFc/m, derive a symbolic expression for this ratio.

We have,



So it would appear that the R3/T2 ratio *should* be constant, and that it’s value is GMsun/4π2.

(b) Using Msun = 2×1030kg, calculate the value of this ratio, and compare it to Kepler’s values. Might help to recall that a Newton N = kg∙m/s2. Is your ratio close to Kepler’s values?

Filling in numbers, and converting AU’s and days,



Yes, it is close!

**Problem 7.** The International Space Station (ISS) orbits Earth roughly 205miles ( = 330km) above the surface. Earth’s radius is roughly 6370km, and its mass is 5.98×1024kg.

(a) Starting from ac = ΣFc/m, calculate the speed of the ISS.

As usual,



(b) Calculate its orbital period. How many times does it orbit Earth per day?

Period is given by:



This would be:

